Inaam Shamma

Quiz I: Math. for the Architects, MTH 111, Spring 2017

Ayman Badawi

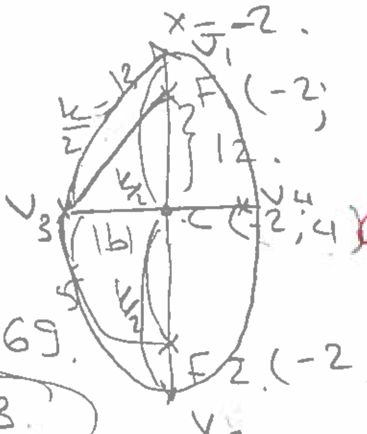
15/15

QUESTION 1. Consider the Ellipse $\frac{(x+2)^2}{25} + \frac{(y-4)^2}{169} = 1$

$\left(\frac{k}{2}\right)^2$

(i) Sketch (rough sketch)

2



$\left(\frac{k}{2}\right)^2 = 169.$

$\frac{k}{2} = 13.$

$\boxed{k = 26}$

(ii) Find the Foci

2

$F_1(-2; 16)$

$F_2(-2; -8)$

$C(-2; +4)$

$b^2 = 25.$

$\boxed{b = 5.}$

hyp² = side² + side²

$169 = 25 + |F_1C|^2$

$|F_1C|^2 = 144.$

$\boxed{F_1C = 12}$

(iii) Find the ellipse-contant k

2

$\boxed{k = 26}$

(iv) Find all 4 vertices.

2

$V_3(-7; 4)$ $V_4(3; 4)$ $V_1(-2; 17)$ $V_2(-2; -9)$

QUESTION 2. Given $(-3, 5)$ is the focus of a parabola with directrix line $x = 9$.

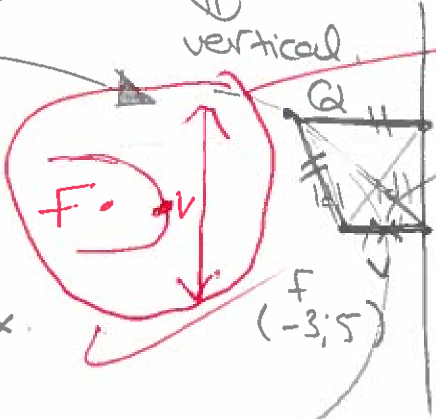
vertical

(i) Sketch (rough sketch)

2

$x = 9$.

graph sketch on the back



(ii) Find the equation of the Parabola.

eq: $4d(x - x_1) = (y - y_1)^2$.

vertix.

3

midpt of $|FB|$ is the vertix.

$x_V = \frac{x_F + x_B}{2} = \frac{-3 + 9}{2} = 3.$

$|FV| = |VB| = |d| = |\Delta x| = |-3 - 3| = |-6| = 6.$

since on the left side

$d < 0.$

$\boxed{d = -6.}$

$4(-6)(x-3) = (y-5)^2$

$\boxed{-24(x-3) = (y-5)^2.}$

~~(iii) If Q is a point on the curve of the parabola. What is the distance between Q and the directrix?~~

2

$|QF| = |QL|$ QL we draw ⊥ to L.

intersect at point E

E (9; ?)

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

iii) find the distance between ~~vertix~~ and directrix.

$|VB| = \sqrt{\Delta x^2} = |\Delta x| = |9 - 3| = 6.$

Salma Allam 71062

Quiz II: MTH 111,Fall 2017

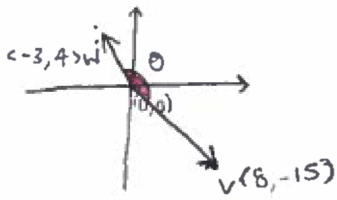
Ayman Badawi

15/15

QUESTION 1. Let $V = <8, -15>$ and $W = <-3, 4>$.

(i) Draw V and W in the XY-plane (start from $(0, 0)$).

1



(ii) Find $V \cdot W$

2

$(-3)(8) + (4)(-15)$

$= -84$

(iii) Find $|V|$ and $|W|$

2

$|V| = \sqrt{(8)^2 + (-15)^2} = 17$

$|W| = \sqrt{(-3)^2 + (4)^2} = 5$

(iv) Find the angle between V and W

2

$\cos\theta = \dfrac{a \cdot b}{|a||b|}$

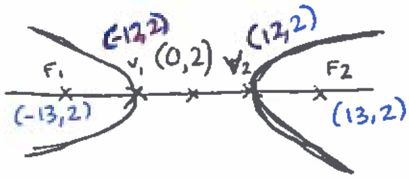
$= \dfrac{-84}{17 \times 5} = -\dfrac{84}{85}$

$\cos^{-1}\left(-\dfrac{84}{85}\right) = 171.2°$

QUESTION 2. Given $\dfrac{x^2}{144} - \dfrac{(y-2)^2}{25} = 1$

(i) Sketch (roughly)

2



(ii) Find the Hyperbola-Constant k

2

$\left(\frac{K}{2}\right)^2 = 144 \quad \frac{K}{2} = 12 \quad K = 24$

(iii) Find the two vertices, i.e., V_1, V_2

2

$V_1 = (0-12, 2) \quad V_1 = (-12, 2)$

$V_2 = (0+12, 2) \quad V_2 = (12, 2)$

(iv) Find the foci, i.e., F_1, F_2

2

$|CF_1| = \sqrt{\left(\frac{k}{2}\right)^2 + b^2}$

$= \sqrt{144 + 25} = 13$

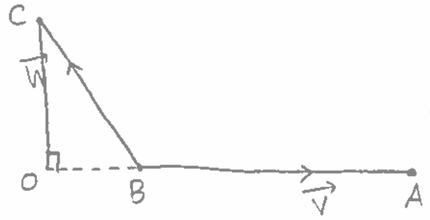
$F_1 = (0-13, 2) \quad F_1 = (-13, 2)$

$F_2 = (0+13, 2) \quad F_2 = (13, 2)$

Faculty information

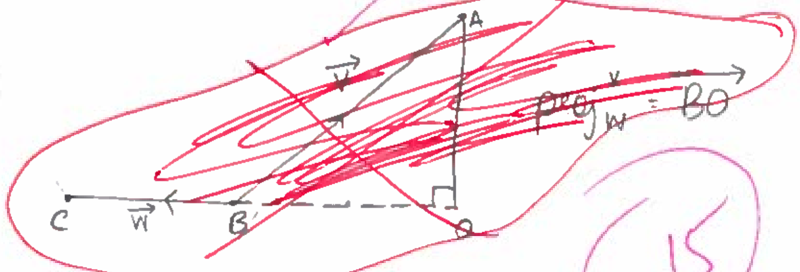
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

i.) a)



$proj_V^W = \overrightarrow{BO}$

~~b)~~



~~$proj_W^V = BO$~~

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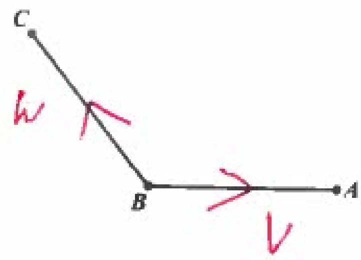
Quiz III, MTH 111, Fall 2017
Ayman Badawi

Name MISA : VAYA , ID number g00075848

Q1. V = ~~AB~~ BA, W = BC. (note ~~AB~~ BA is a horizontal directed line segment)



a) Draw the projection vector of W over V
~~b) Draw the projection of V over W.~~

4

Q2. V = <- 3, -4, 0>, W = <-2, 1, 2> .

a) Find the projection vector of W over V, name it P.

$$P = proj_V^W = \frac{W \cdot V}{|V|^2} V$$

$$= \frac{6-4+0}{25} \langle -3, -4, 0 \rangle = \frac{2}{25} \langle -3, -4, 0 \rangle$$

4

$$= \left\langle \frac{-6}{25}, \frac{-8}{25}, 0 \right\rangle //$$

b) Find |P|

$$|P| = |proj_V^W| = \frac{|W \cdot V|}{|V|} = \frac{2}{\sqrt{25}} = \frac{2}{5} //$$

3

c) Find two vectors L and U such that W = U + L, where L is parallel to V and u is perpendicular to V.

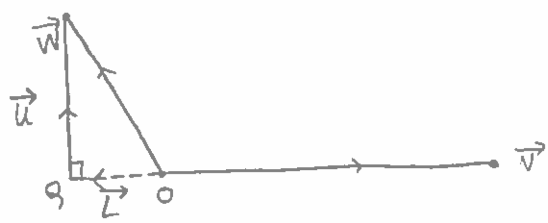
$$W = U + L$$

$$\vec{L} = \overrightarrow{OQ} = proj_V^W$$

4

$$= \frac{W \cdot V}{|V|^2} V \text{ (from 2.) a.))}$$

$$= \left\langle \frac{-6}{25}, \frac{-8}{25}, 0 \right\rangle //$$



$\langle -3, -4, 0 \rangle \cdot$
$\langle \frac{-44}{25}, \frac{33}{25}, 2 \rangle$

$\frac{-2+6}{25}$

$\frac{-50+6}{25} = \frac{-44}{25}$

$\frac{25}{8}$
33

$= \frac{44 \times 3}{25} - \frac{4 \times 33}{25}$

$= \frac{132}{25} - \frac{132}{25} = 0 //$

$$\vec{u} = W - L$$

$$= \langle -2, 1, 2 \rangle + \left\langle \frac{+6}{25}, \frac{+8}{25}, 0 \right\rangle$$

$$= \left\langle \frac{-44}{25}, \frac{33}{25}, 2 \right\rangle //$$

Quiz Four: MTH 111, Fall 2017

Ayman Badawi

15/15

QUESTION 1. Find a parametric equations of the line that passes through (1, 6, 9) and (0, 4, −1)

① $L_D : \langle 0-1, 4-6, -1-9 \rangle$

$= \langle -1, -2, -10 \rangle$

② $L : (1,6,9) + t\langle -1,-2,-10 \rangle$

$= (1+-t,\ 6+-2t,\ 9+-10t)$

$$\begin{cases} x = 1-t \\ y = 6-2t \\ z = 9-10t \end{cases} \quad t = \mathbb{R}$$

5

QUESTION 2. Find a parametric equations of the line that has directional vector $D = \langle 3, -4, 8 \rangle$ and it passes through (2, −6, 7)

$L : (2,-6,7) + t\langle 3,-4,8 \rangle$

$= (2+3t,\ -6+-4t,\ 7+8t)$

$$\begin{cases} x = 2+3t \\ y = -6-4t \\ z = 7+8t \end{cases} \quad t = \mathbb{R}$$

5

QUESTION 3. Does $L_1 : x = 2t+1, y = -4t+6, z = 3t+2$ $(t \in R)$ intersect $L_2 : x = 4w+1, y = w-12, z = 4w+6$ $(w \in R)$? If yes, then find the intersection point.

L_1:
$$\begin{cases} x = 2t+1 \\ y = -4t+6 \\ z = 3t+2 \end{cases} \quad t = \mathbb{R}$$

L_2:
$$\begin{cases} x = 4w+1 \\ y = w-12 \\ z = 4w+6 \end{cases} \quad w = \mathbb{R}$$

① FIND T & W

$2t+1 = 4w+1$

$-4t+6 = w-12$

↓

$2t-4w = 1-1$

$-4t-w = -12-6$

↓

$2t-4w = 0$

$-4t-w = -18$

$$t = \frac{\begin{vmatrix} 0 & -4 \\ -18 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ -4 & -1 \end{vmatrix}} \qquad w = \frac{\begin{vmatrix} 2 & 0 \\ -4 & -18 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ -4 & -1 \end{vmatrix}}$$

$t = \dfrac{0-(72)}{-2-(16)}$ $\qquad w = \dfrac{-36-0}{-2-16}$

$t = \dfrac{-72}{-18}$ $\qquad w = \dfrac{-36}{-18}$

$t = 4$ $\qquad w = 2$

CHECK: z OF L_1 $\overset{?}{=}$ z OF L_2

$3t+2 \overset{?}{=} 4w+6$

$(3)(4)+2 \overset{?}{=} 4(2)+6 \rightarrow 14 = 14$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

② L_1 INTERSECT L_2

YES!

③ FIND INTERSECTION POINT → (9, −10, 14)

$x = 2(4)+1 = 9$

$y = -4(4)+6 = -10$

$z = 3(4)+2 = 14$

5

Quiz 6: MTH 111, Fall 2017

Ayman Badawi

15/15

QUESTION 1. Let $Q_1 = (1, 3, 4)$, $Q_2 = (-4, 1, 8)$, and $Q_3 = (-3, 4, 10)$.

a) Are Q_1, Q_2, Q_3 co-linear? EXPLAIN

$\overrightarrow{Q_1Q_2} = \langle -5, -2, 4 \rangle$

$\overrightarrow{Q_1Q_3} = \langle -4, 1, 6 \rangle$

$$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -2 & 4 \\ -4 & 1 & 6 \end{vmatrix}$$

4/4

$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = -2(6) - 1(4)\,\hat{i} + 14\hat{j} - 13\hat{k}$

$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = -16\hat{i} + 14\hat{j} - 13\hat{k}$

→ They are not collinear because the cross-product is not a zero vector.

b) If the answer in (a) is NO, then find the area of the triangle determined by Q_1, Q_2, Q_3.

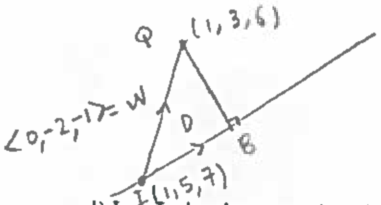
$\text{Area}\,\Delta = \frac{1}{2}\left|\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}\right|$

$\text{Area}\,\Delta = \frac{1}{2}\sqrt{16^2 + 14^2 + 13^2} = \frac{3\sqrt{69}}{2}$ units^2

3/3

c) Let $L : x = 2t + 1, y = -4t + 5, z = 2t + 7$ $(t \in R)$. Then the points $Q = (1, 3, 6)$ is not on L. Find $|QL|$ [You must use the idea of CROSS PRODUCT to find $|QL|$ as we did on Tuesday.]

$D = \langle 2, -4, 2 \rangle$



$$|\overrightarrow{BQ}| = \frac{|W \times D|}{|D|} = \frac{\sqrt{8^2 + 2^2 + 4^2}}{\sqrt{2^2 + 4^2 + 2^2}} = \frac{\sqrt{14}}{2}$$

$$W \times D = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & -1 \\ 2 & -4 & 2 \end{vmatrix} = -8\hat{i} - 2\hat{j} + 4\hat{k}$$

4/4

$\frac{\sqrt{84}}{\sqrt{24}}$

d) Let L_1 be the same L as in (c). Let $L_2 : x = w + 1, y = -3w + 2, z = w + 4$ $(w \in R)$. Convince me that L_1 is not parallel to L_2.

$L_1 : \begin{cases} x = 2t + 1 \\ y = -4t + 5 \, ; \, t \in \mathbb{R} \\ z = 2t + 7 \end{cases}$ → $D_1 = \langle 2, -4, 2 \rangle$

$L_2 : \begin{cases} x = w + 1 \\ y = -3w + 2 \, ; \, w \in \mathbb{R} \\ z = w + 4 \end{cases}$ → $D_2 = \langle 1, -3, 1 \rangle$

4/4

→ $D_2 \neq cD_1$

D_1 is not parallel to D_2 (directional vectors are not parallel)

⇒ The lines are not parallel

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name [illegible], ID [illegible]

Quiz 7: MTH 111, Fall 2017

Ayman Badawi

$\frac{15}{15}$

QUESTION 1. Let $Q_1 = (1, 0, 2), Q_2 = (-2, 0, 2), Q_3 = (1, 2, 6)$. Find the equation of the plane determined by Q_1, Q_2, Q_3.

$\overrightarrow{Q_1Q_2} = \langle -3, 0, 0 \rangle$

$\overrightarrow{Q_1Q_3} = \langle 0, 2, 4 \rangle$

$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

$= \begin{vmatrix} i & j & k \\ -3 & 0 & 0 \\ 0 & 2 & 4 \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 \\ 2 & 4 \end{vmatrix} i - \begin{vmatrix} -3 & 0 \\ 0 & 4 \end{vmatrix} j + \begin{vmatrix} -3 & 0 \\ 0 & 2 \end{vmatrix} k$

$= (0-0)i - (-12-0)j + (-6-0)k = 0i + 12j - 6k = \langle 0, 12, -6 \rangle$

$P \to \langle 0, 12, -6 \rangle \cdot \langle x-1, y-0, z-2 \rangle = 0$

$0(x-1) + 12(y-0) + -6(z-2) = 0$

$0x + 12y - 6z + 12 = 0$

$12y - 6z + 12 = 0$

$12y - 6z = -12$

$\frac{5}{5}$

QUESTION 2. The plane $x + 2y + 3z = 26$ intersects the line $L : x = t, y = t + 1, Z = 3t$ ($t \in R$) in exactly one point, say Q. Find Q.

$P_1 \to x + 2y + 3z = 26$

$L \to \left. \begin{array}{l} x = t \\ y = t + 1 \\ z = 3t \end{array} \right] t = \mathbb{R}$

$t + 2(t+1) + 3(3t) = 26$

$t + 2t + 2 + 9t = 26$

$12t + 2 = 26$

$\frac{12t}{12} = \frac{24}{12} \to t = 2$

$\boxed{Q \to (2, 3, 6)}$

$\frac{4}{4}$

QUESTION 3. a) The plane $2x + 4y + 6z = 18$ is parallel to the plane in question (2). Find the distance between the two planes.

$P_1 \to x + 2y + 3z = 26$

$P_2 \to 2x + 4y + 6z = 18$

$P_1 \parallel P_2$

$N_1 \to \langle 1, 2, 3 \rangle$

$E_1 \to x + 2y + 3z - 26$

$a \to (0, 0, 3)$

$|P_1P_2| = |QP_1| = \frac{|Q \text{ in } E_1|}{|N_1|}$

$= \frac{|0 + 2(0) + 3(3) - 26|}{\sqrt{1^2 + 2^2 + 3^2}}$

$= \frac{|9 - 26|}{\sqrt{1 + 4 + 9}} = \frac{|-17|}{\sqrt{14}} = \frac{17}{\sqrt{14}}$

$\frac{3}{3}$

b) Can we draw the vector $< -13, 2, 3 >$ inside the plane given in (a)? explain

$N_2 \to \langle 2, 4, 6 \rangle$

IF $V \perp N_2 = V \cdot N_2 = 0$, THEN YES.

$V \cdot N_2 = \langle -13, 2, 3 \rangle \cdot \langle 2, 4, 6 \rangle$

$= -26 + 8 + 18$

$= -26 + 26$

$= 0 \therefore V \perp N_2$. YES V CAN BE DRAWN IN THE PLANE.

$\frac{3}{3}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Quiz 8: MTH 111, Fall 2017

Ayman Badawi

15/15

QUESTION 1. Let $f(x) = 2x^3 + 3x^2 - 36x + 1$

(i) Find the critical values of $f(x)$

$f'(x) = 6x^2 + 6x - 36$

$0 = 6x^2 + 6x - 36$

$0 = 6(x^2 + x - 6)$

4

$x = 2$

$x = -3$

(ii) Find the equation of the tangent to the curve of $f(x)$ at each critical value of $f(x)$.

$f(2) = 2(2)^3 + 3(2)^2 - 36(2) + 1$
$= -43$

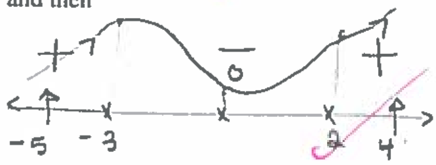
3

$f(-3) = 2(-3)^3 + 3(-3)^2 - 36(-3) + 1$
$= 82$

The line is y = -43
The line is y = 82

(iii) Find the sign of $f'(x)$ and then

4



$f'(-5) = 6(-5)^2 + 6(-5) - 36$
$= +84$

$f'(0) = 6(0)^2 + 6(0) - 36$
$= -36$

$f'(4) = 6(4)^2 + 6(4) - 36$
$= +84$

a. For what values of x does $f(x)$ increase?

$f(x)$ increases at $(-\infty, -3) \cup (2, +\infty)$

2

b. For what values of x does $f(x)$ decrease?

$f(x)$ decreases at $(-3, 2)$

c. Find the local min. and local max. values of $f(x)$

2

$f(-3) = 82 \rightarrow$ local Max

$f(2) = -43 \rightarrow$ local Min
$(2, -43)$

max value of f(x) or of y is 82
and it occurs when x = -3

Min. value of f(x) or of y is -43 and it occurs when x = 2

d. Sketch roughly $f(x)$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name Haya Alshamsi, ID g00075530.

Quiz 9: MTH 111, Fall 2017

Ayman Badawi

13/15

QUESTION 1. Find $f'(x)$

(i) $f(x) = \sqrt{3x} + 2x + 3$.

$f(x) = (3x)^{1/2} + 2x + 3$

$f'(x) = \frac{\sqrt{3}}{2\sqrt{x}} + 2$.

3

(ii) $f(x) = (2+x)^5 + \frac{3}{x^7} + 4x + 2$; $f(x) = (2+x)^5 + 3x^{-7} + 4x + 2$

$f'(x) = 5(2+x)^4 - 21x^{-8} + 4$

$f'(x) = 5(2+x)^4 - \frac{21}{x^8} + 4$

(iii) Given $f(x) = k(2x^3 + x)$ and $k'(3) = -4$. Find $f'(1)$.

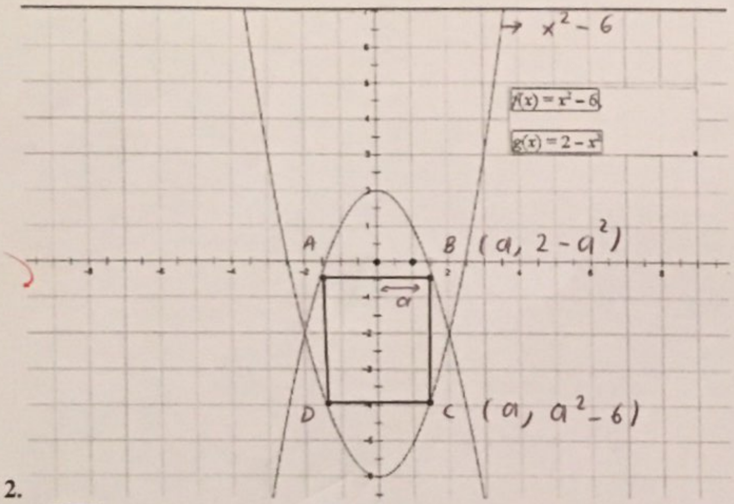
$f'(x) = (6x^2+1)\,k'(2x^3+1)$

$f'(1) = 7\,k'(3) = 7(-4) = -28$

$f'(x) = k'(2x^3+x)(6x^2+1)$

$k'(3)(7) =$

$-4(7) = -28$



Find the length and the width of the rectangle with maximum area and it can be drawn between f(x) = x^2 - 6 and g(x) = 2 - x^2. (see picture)

QUESTION 2.

$A = L \times W$

$L = \overline{AB} = 2a$

$W = \overline{BC} = 2 - a^2 - (a^2 - 6)$

$W = 2 - a^2 - a^2 + 6$

$W = 8 - 2a^2$

3/4

$A = 2a(8 - 2a^2)$

$A = 16a - 4a^3$

$A' = 16 - 12a^2$

$A' = 0$

$16 = 12a^2$

$a^2 = \frac{16}{12}$

$a = \pm\sqrt{\frac{4}{3}}$; $\boxed{a > 0}$

$\Rightarrow \boxed{a = \sqrt{\frac{4}{3}}}$

3

$A'' = -24a$

$A'' = -24\sqrt{\frac{4}{3}} < 0$.

$\Rightarrow$ The Area is maximum. when $x = +\sqrt{\frac{4}{3}}$.

The length $= \overline{AB} = 2\sqrt{\frac{4}{3}} = \frac{4\sqrt{3}}{3}$

The width $\overline{BC} = 8 - 2\left(\frac{4}{3}\right) = 8 - \frac{8}{3} = \frac{16}{3}$

Maximum Area $= \frac{64\sqrt{3}}{9}$

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E-mail: abadawi@aus.edu, www.ayman-badawi.com